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Abstract

Superimposed training (ST) technique can be used at primary users 19 transmitters to improve parameter estimation tasks (e.g. channel estimation) at primary users 19 receivers at the time the total available bandwidth is used for data transmission. The exploitation of the ST sequence in the context of cognitive radio networks leads to a signiï¬□cant increase in the detection performance of secondary users operating in the very low signal-to-noise ratio region. Hence, a smaller number of samples are required for sensing. In this paper, the performance of ST-based spectrum sensing in a cooperative centralized cognitive radio network with soft-decision fusion is studied. Furthermore a throughput analysis is carried out to quantify the beneï¬□ts of using ST in the context of cognitive radio for both primary and secondary users.

Performance analysis of superimposed training-based cooperative spectrum sensing

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***Abstract*—Superimposed training (ST) technique can be used at primary users' transmitters to improve parameter estimation tasks (e.g. channel estimation) at primary users' receivers at the time the total available bandwidth is used for data transmission. The exploitation of the ST sequence in the context of cognitive radio networks leads to a significant increase in the detection performance of secondary users operating in the very low signal-to-noise ratio region. Hence, a smaller number of samples are required for sensing. In this paper, the performance of ST-based spectrum sensing in a cooperative centralized cognitive radio network with soft-decision fusion is studied. Furthermore a throughput analysis is carried out to quantify the benefits of using ST in the context of cognitive radio for both primary and secondary users.**

***Index Terms*—Spectrum sensing, cooperative, cognitive radio, superimposed training.**

I. Introduction

Future wireless networks and communications will demand more bandwidth available to support the increasing number of users and their high throughput requirements. For primary users (PUs), i.e. those users with access to a licensed frequency band, extra bandwidth can be obtained based on the fact that wireless digital communications systems employ pilot symbols for parameter estimation tasks. Traditionally, these pilots are inserted in different time slots than data symbols, thus reducing the available bandwidth for data transmission. This is known in the literature as time domain multiplexed training (TDMT). An alternative technique to improve the spectral efficiency, is to superimpose (i.e. add) pilot symbols to data symbols prior transmissions of PUs. This is known in the literature as superimposed training (ST) technique [1]. ST has the advantage of increasing bandwidth of PUs at the time better performance of parameter estimation tasks (e.g. channel estimation or synchronization) are attained at PU's receivers, which is a feature of great interest in future wireless communications systems. Therefore, ST has been actively studied in recent years in this context [2], along with the associated cost that using ST at PUs' transmitters implies an increase in signal processing and infrastructure complexity. Additionally to the bandwidth a PU can gain with the use of ST transmitters, more radio spectrum can be obtained based on the fact that some PUs do not use their allocated frequency band at some time or geographic location. Hence, cognitive radio (CR) technology is envisioned as a solution to improve the radio spectrum usage in an opportunistic manner [3]. In interweave CR networks, unlicensed radio spectrum users (i.e. secondary users) must sense one or more frequency bands to determine if they are being used or not by the primary user to whom were allocated. If a frequency band is free, then a secondary user (SU) can transmit data opportunistically. This process is called spectrum sensing [4] and it is considered the most important task in interweave CR networks. Reducing the spectrum sensing time is a major concern in interweave CR, since more time will be available for opportunistic transmission of a secondary user and therefore higher throughput. Indeed, in [5] the trade-off between sensing time and achievable throughput is studied in the context of CR. It is shown that an optimum sensing time that maximizes the secondary user's throughput exist. Thus, results are shown for energy detection (ED) based spectrum sensing technique, which is the simplest technique to perform spectrum sensing. However, ED does not exploit the fact that many wireless communications systems might include pilot symbols in their signals for parameter estimation purposes, which can be used to improve the performance of spectrum sensing in terms of probabilities of false alarm and detection. In [6] and [7] spectrum sensing algorithms are proposed considering TDMT PUs' signals. Results quantify the detection gain that can be achieved if SUs exploit the knowledge of pilot symbols in the received signal. Thus, in [8] the trade-off problem between sensing and training is analyzed for TDMT-based spectrum sensing in CR. Furthermore, in [9] a PU's signal that adds a pilot symbol to the data symbol is considered in the context of detection of digital television (DTV) signals. Moreover, ST-based spectrum sensing for cognitive radio is considered in [10] for a single SU over noisy channels. It is shown the significant reduction in number of samples needed in the spectrum sensing stage if PUs employ ST transmitters and the high probability of false alarm that can be achieved by SUs. Thus showing that ST is a benefit for both PUs and SUs. In this paper, however, a cooperative centralized cognitive radio network is considered with soft-decision fusion for SUs operating in the very low signal-to-noise ratio (SNR) that employ ST-based spectrum sensing. Hence, closed-form expressions for global probabilities of false alarm and detection are obtained. Results quantify the significant reduction in number of samples required in the sensing stage and the higher achievable throughput attained by ST-based spectrum sensing even for a single user, in comparison to the ED. Moreover, it is shown that the SUs' throughput gain depends not only in the number of samples required in sensing but also in the training-to-information ratio (TIR) in ST-based PU's communications systems.

II. System model and hypothesis test

Consider a centralized cognitive radio network (CRN) consisting of K cooperative SUs and a fusion center (FC) as shown in Fig. 1. In the first instance, it is considered that all SUs sense a PU's frequency band. Assume that the PU's transmitter implements the ST technique to provide the PU's receiver with a better channel estimation at the same time the whole available bandwidth is used for data transmission. Hence, the ST-based PU's signal is given by the addition of a low power periodic training sequence to the data sequence. Moreover, in order to reduce interferences from SU's transmissions, assume that the PU previously shares the ST sequence with SUs, which are block synchronized with the PU. Furthermore, assume that each SU individually performs local sensing at the PU's frequency band and sends its test statistic to the FC through a control channel. Then, based on soft-decision fusion the FC determines the presence (i.e. hypothesis $\mathcal{H}_1$) or absence (i.e. hypothesis $\mathcal{H}_0$) of the PU's signal. Thus, individually, a SU must decide between the following two hypothesis based on the received signal samples at their receivers:

$$\begin{aligned} \mathcal{H}_0 &: x_k[n] = w_k[n] \ , \\ \mathcal{H}_1 &: x_k[n] = (d[n] + t[n]) + w_k[n] \ , \end{aligned} \tag{1}$$

where $1 \le k \le K$ and $0 \le n \le N-1$, N represents the total number of samples used in the sensing stage. Additionally, each sample in both PU's data sequence $\{d[n]\}_{n=0}^{N-1}$ and noise sequence $\{n_k[n]\}_{n=0}^{N-1}$ are zero-mean independent and identically distributed (i.i.d.) circularly symmetric complex random variables with variance σ_d^2 and σ^2, respectively. Moreover, the periodic training sequence $\{t[n]\}_{n=0}^{N-1}$ (with period P) is deterministic and $|t[n]|^2 = 1$ for $n \mod P = 0$ and $|t[n]|^2 = 0$ otherwise. Furthermore, the TIR is defined as $\alpha = \sigma_t^2/\sigma_d^2$, where $\sigma_t^2 = \sum_{n=0}^{N-1} |t[n]|^2$ is the average power in the training sequence and $\sigma_d^2 = \sum_{n=0}^{N-1} |d_k[n]|^2$ is the average power in the data sequence, which without loss of generality is assumed to be unitary.

III. Single node ST-based spectrum sensing

Making the correct choice between hypotheses in (1) can be a challenging task, especially in a very low signal-to-noise ratio (SNR) operating scheme. Hence, two errors can be made:

i) Deciding $\mathcal{H}_0$ when $\mathcal{H}_1$ is true, thus causing interference to PU's transmissions. When this error occurs, it is said that there is a false alarm.
ii) Deciding $\mathcal{H}_1$ when $\mathcal{H}_0$ is true, thus loosing opportunities for data transmissions. When this error occurs, it is said that there is a missed-detection.

Thus, the detection performance of a spectrum sensing technique can be characterized by the probability of false alarm and the probability of missed-detection (or alternatively, the probability of detection, which is the probability of correctly decide $\mathcal{H}_1$). It is of great interest that the method used for spectrum sensing performs with the higher probability of detection and the lower probability of false alarm. This last might not be totally possible given the trade-off between theses probabilities: reducing the probability of false alarm reduces the probability of detection, and vice versa. Therefore, in order to improve the detection performance by exploiting the known training sequence at SUs, it is used the Neyman-Pearson detector for ST-based PU's signals (called STD) [10]. Hence, the test statistic at each secondary user is given by

$$T_{\mathrm{st},k}(x_k) = \sum_{n=0}^{N-1} |x_k[n]|^2 + \frac{2}{\gamma} \Re \left\{ \sum_{n=0}^{N-1} x_k^*[n] t[n] \right\}, \tag{2}$$

where $\gamma = \sigma_d^2/\sigma^2$ is the received signal-to-noise ratio (SNR) of the PU measured at the k-th SU's receiver.

A. Performance metrics

For a single SU, the probability of false alarm, P_{fa}, of the STD is given by [10]

$$P_{fa}(\lambda, \alpha, N) = \mathcal{Q}\left(\frac{\lambda - N\sigma^2}{\sigma^2 \sqrt{N\left[(2\alpha/\gamma)+1\right]}} \right), \tag{3}$$

where $\mathcal{Q}(\cdot)$ is the complementary cumulative distribution function of a standard Gaussian distribution [11] and λ is the detection threshold. Moreover, the probability of detection is given by [10]

$$P_d(\lambda, \alpha, N) = \mathcal{Q}\left(\frac{\lambda - N\left[2\sigma^2\alpha + \upsilon\right]}{\sqrt{N\left[\upsilon^2 + \frac{2\alpha\sigma^2}{\gamma}\upsilon\right]}} \right). \tag{4}$$

where $\upsilon = \sigma_d^2 + \sigma^2$. Note that besides both P_{fa} and P_d depend on the threshold and the number of samples used in the sensing stage they also depend on the TIR used by the PU in the ST transmitter. Thus, for a target probability of detection, $\bar{P}_d$, the detection threshold is found from (4), i.e.

$$\lambda = \mathcal{Q}^{-1}(\bar{P}_d)\sqrt{N\left[\upsilon^2 + \frac{2\alpha\sigma^2}{\gamma}\upsilon\right]} + N\left[2\sigma^2\alpha + \upsilon\right]. \tag{5}$$

Therefore, for a target probability of detection, $\bar{P}_d$, the probability of false alarm is:

$$\begin{aligned} P_{fa}(\alpha, N) = \mathcal{Q}\Bigg(&\frac{N\left[2\sigma^2\alpha + \upsilon\right] - N\sigma^2}{\sigma^2\sqrt{N\left[(2\alpha/\gamma)+1\right]}} \\ &+ \frac{\mathcal{Q}^{-1}(\bar{P}_d)\sqrt{N\left[\upsilon^2 + \frac{2\alpha\sigma^2}{\gamma}\upsilon\right]}}{\sigma^2\sqrt{N\left[(2\alpha/\gamma)+1\right]}} \Bigg). \end{aligned} \tag{6}$$

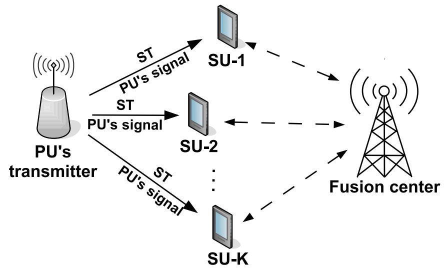

Fig. 1. Centralized cooperative spectrum sensing scenario with one primary user (PU) implementing ST technique, K secondary users (SUs) and one fusion center (FC).

IV. Cooperative ST-based Spectrum Sensing

ST-based spectrum sensing is analyzed next in a centralized cooperative CRN with soft decision fusion. Hence, after each SU performs ST-based spectrum sensing and makes a decision on the absence/presence of a PU in a frequency band of interest, all SUs report their test statistic, $T_{\mathrm{ST},k}(x_k)$ given by (2), to the FC, which combines them to make a global decision. Hence, the FC decides $\mathcal{H}_1$ if

$$T_{\mathrm{ST}} = \sum_{k=1}^{K} T_{\mathrm{st},k}(x_k) > \Lambda, \tag{7}$$

where Λ is the global detection threshold. For large N, the detection performance of (7) can be obtained by invoking the CLT. Hence, when $\mathcal{H}_0$ is true T_{ST} is approximated by a Gaussian distributed random variable (r.v.) with mean, μ_0, given by

$$\mu_0 = \sum_{k=1}^{K} \mathrm{E}\left[T_{\mathrm{st},k}(x_k)|\mathcal{H}_0\right] = KN\sigma^2 \tag{8}$$

and variance, σ_0^2, given by

$$\begin{aligned}\sigma_0^2 =& \sum_{k=1}^{K} \mathrm{var}\left[T_{\mathrm{st},k}(x_k)|\mathcal{H}_0\right] \\ =& KN\sigma^4\left(\frac{2\alpha}{\gamma}+1\right).\end{aligned} \tag{9}$$

Hence, the global probability of false alarm, P_{FA} can be expressed as:

$$P_{\mathrm{FA}}(\Lambda,\alpha,N) = \mathcal{Q}\left(\frac{\Lambda - KN\sigma^2}{\sigma^2\sqrt{KN\left[\frac{2\alpha}{\gamma}+1\right]}}\right). \tag{10}$$

On the other hand, when $\mathcal{H}_1$ is true, T_{ST} is approximated to a Gaussian distribution with mean, μ_1, given as follows:

$$\begin{aligned}\mu_1 =& \sum_{k=1}^{K} \mathrm{E}\left[T_{\mathrm{st},k}(x_k)|\mathcal{H}_1\right] \\ =& KN\left[2\sigma^2\alpha + \upsilon\right]\end{aligned} \tag{11}$$

and variance, σ_1^2, given as follows:

$$\begin{aligned}\sigma_1^2 =& \sum_{k=1}^{K} \mathrm{var}\left[T_{\mathrm{st},k}(x_k)|\mathcal{H}_1\right] \\ =& KN\left[\upsilon^2 + \frac{2\alpha\sigma^2}{\gamma}\upsilon\right].\end{aligned} \tag{12}$$

Hence, the global probability of detection, P_{D}, can be expressed as:

$$P_{\mathrm{D}}(\Lambda,\alpha,N) = \mathcal{Q}\left(\frac{\Lambda - KN\left[2\sigma^2\alpha+\upsilon\right]}{\sqrt{KN\left[\upsilon^2+\frac{2\alpha\sigma^2}{\gamma}\upsilon\right]}}\right). \tag{13}$$

It is worth noting that P_{FA} and P_{D} are also functions of the TIR α. Thus, for a target global probability of detection, $\bar{P}_{\mathrm{D}}$, the detection threshold is given by

$$\begin{aligned}\Lambda =& \mathcal{Q}^{-1}(\bar{P}_{\mathrm{D}})\sqrt{KN\left[\upsilon^2+\frac{2\alpha\sigma^2}{\gamma}\upsilon\right]} \\ &+ KN\left[2\sigma^2\alpha+\upsilon\right].\end{aligned} \tag{14}$$

Hence, for a target probability of detection $\bar{P}_{\mathrm{D}}$, the probability of false alarm is

$$\begin{aligned}P_{\mathrm{FA}}(\alpha,N) =& \mathcal{Q}\left(\frac{\mathcal{Q}^{-1}(\bar{P}_{\mathrm{D}})\sqrt{\upsilon^2+\frac{2\alpha\sigma^2}{\gamma}\upsilon}}{\sigma^2\sqrt{\frac{2\alpha}{\gamma}+1}}\right. \\ &\left. + \sqrt{KN}\frac{\sigma^2\left(2\alpha-1\right)+\upsilon}{\sigma^2\sqrt{\frac{2\alpha}{\gamma}+1}}\right).\end{aligned} \tag{15}$$

Thus, from (15) it can be seen that for a given value of $\bar{P}_{\mathrm{D}}$, the global probability of false alarm depends on the value of α and N.

V. Analysis of Throughput

In a cooperative cognitive radio scenario in which SUs perform ST-based spectrum sensing periodically, the SU's throughput depends on the number of samples used in the sensing stage and the TIR selected by the PU (i.e. N and α). The average SU's throughput is defined in [5] as follows:

$$\begin{aligned}B(N,\alpha) =& \left(1-\frac{N}{M}\right)C_0\left(1-P_{\mathrm{FA}}(\Lambda,\alpha,N,K)\right)P(\mathcal{H}_0) \\ &+ \left(1-\frac{N}{M}\right)C_1\left(1-P_{\mathrm{D}}(\Lambda,\alpha,N,K)\right)P(\mathcal{H}_1),\end{aligned} \tag{16}$$

where M is the total length of the cognitive radio frame, $P(\mathcal{H}_0)$ is the prior probability that the PU is inactive, $P(\mathcal{H}_1)$ is the prior probability that the PU is active. Moreover, C_0 is SU's throughput when the SU operates under $\mathcal{H}_0$ defined as:

$$C_0 = \log_2\left(1+\frac{P_s}{\sigma_r^2}\right) \tag{17}$$

where P_s is the received power of the SU and σ_r^2 is the noise power at the SU's receiver. Additionally, C_1 is the SU's throughput when it operates under $\mathcal{H}_0$, given by:

$$C_1 = \log_2(1 + P_s/(P_p + \sigma_r^2)) \tag{18}$$

where P_p is the interference power of the PU at the SU's receiver. From (16) it can be seen that increasing N decreases the available time for SU's data transmission. Thus, decreases the throughput. On the other hand, increasing N decreases also the probability of false alarm, which increases the throughput. Therefore, there is an optimal number of samples that maximizes the SU's throughput. Given that $C_0 > C_1$, the first term in (16) contributes more to the threshold. Thus, the SU's throughput can be approximated as [5]:

$$\tilde{B}(N, \alpha) = \left(1 - \frac{N}{M}\right) C_0 \left(1 - P_{\mathrm{FA}}(\Lambda, \alpha, N, K)\right) P(\mathcal{H}_0).$$

This approximation is used for the results shown in next section.

VI. Results

It is considered a centralized cooperative spectrum sensing scenario with the soft-decision rule given by (7), implemented in the FC. Additionally, it is considered that the PU utilizes the ST technique and the SU have prior knowledge of the training sequence. Moreover, it is assumed that P_s/σ_r^2 is the same at each SU. The target global probability of detection is set to $\bar{P}_{\mathrm{D}} = 0.9$, $\gamma = -20$dB and the total length of the cognitive radio frame is $M = 600000$. Firstly, Fig. 2 shows a comparison of the global probability of false alarm against number of samples N, obtained via Monte Carlo simulations with 10000 trials and the theoretical results from (15). The total number of SU is equal to $K = 6$ and results are shown for different values of TIR, α. Furthermore in this figure, theoretical results for the energy detector are displayed to show the gain in P_{FA} when ST-based spectrum sensing is used with small values of TIR. Thus, it is shown that the number of samples required for sensing significantly decreases with the increase of α. However, it is worth noting that even for small values of α much less samples are needed in the sensing stage when compared to the ED and also much lower values of P_{FA} are attained. In Fig. 3 it is displayed the achievable throughput against number of samples N for different numbers of SUs and a TIR value of $\alpha = 0.1$. As expected, the SU's throughput increases with the increase of K and in comparison to the ED the SU achieve higher levels of throughput. Next, Fig. 4 shows the SU's throughput against number of samples N for different values of TIR and it is compared with the SU's throughput achieved by the energy detector. It can be observed that there is an optimal number of samples, which is significantly smaller than that of the ED for greater values of TIR. Thus, there is a gain in throughput when exploiting the ST sequence at SUs in comparison to the ED. Finally, Fig. 5 shows the optimal number of samples N that maximizes the SU's throughput against different number of cooperating SUs K and for different values of TIR α. It can be observed in this

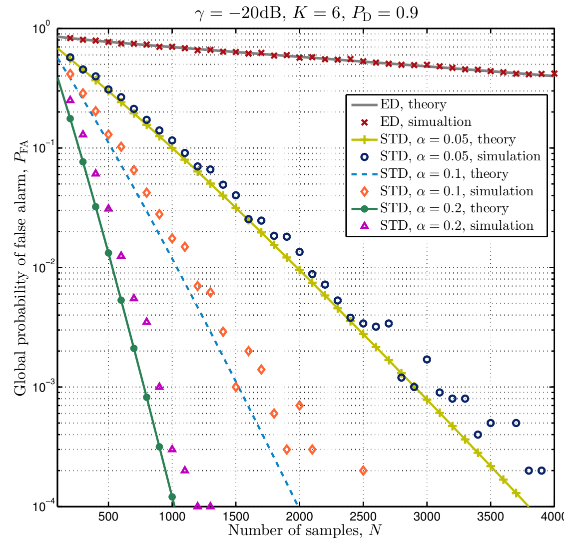

Fig. 2. Global probability of false alarm against number of samples, N, for different values of training-to-information ratio, α.

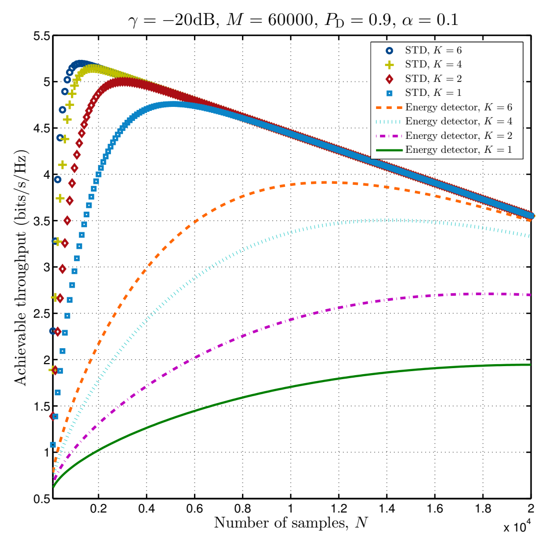

Fig. 3. Achievable throughput for cooperative spectrum sensing with $\alpha = 0.1$ against number of samples for different numbers of cooperative SU, K.

graphic that the optimal number of samples that maximizes the throughput is considerable reduced when exploiting the ST technique. For example, for $K = 6$, and $\alpha = 0.05$, the optimal N for the STD is 2000, approximately, whereas for the energy detector the optimal N is approximately 15000, which is a significant difference.

VII. Conclusions

The performance of ST-based spectrum sensing was analyzed in a centralized cognitive radio network with soft-decision fusion in a noisy environment. Hence, closed-form

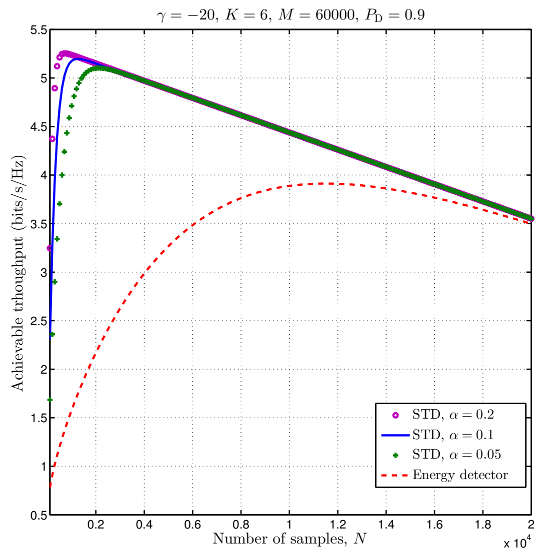

Fig. 4. Achievable throughput for cooperative spectrum sensing with $K = 6$ users against number of samples for different values of training-to-information ratio, α.

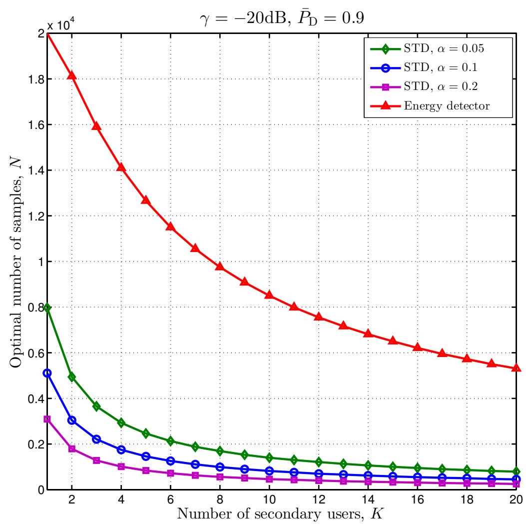

Fig. 5. Optimal number of samples N against different number of cooperative SUs, K.

expressions were obtained for the global probabilities of false alarm and detection. Furthermore, the optimal number of samples in the ST-based sensing stage that maximizes the secondary user's throughput was found. Simulation results quantified the significant detection gained by the ST-based spectrum sensing algorithm in comparison to the energy detector when SUs cooperate in a cognitive radio network. Moreover, it was shown that the maximum achievable throughput is also a function of the TIR value at the PU's transmitter, which is selected by the PU. Further studies are needed to evaluate the performance of ST-based spectrum sensing over fading channels.

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