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Abstract

Consider the problem of scheduling a set of sporadically arriving tasks on a uniform multiprocessor with the goal of meeting deadlines. A processor p has the speed Sp. Tasks can be preempted but they cannot migrate between processors. We propose an algorithm which can schedule all task sets that any other possible algorithm can schedule assuming that our algorithm is given processors that are three times faster..

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Abstract

Consider the problem of scheduling a set of sporadically arriving tasks on a uniform multiprocessor with the goal of meeting deadlines. A processor p has the speed S_p. Tasks can be preempted but they cannot migrate between processors. We propose an algorithm which can schedule all task sets that any other possible algorithm can schedule assuming that our algorithm is given processors that are three times faster.

1. Introduction

Consider the problem of preemptive scheduling of a set τ of n sporadically arriving tasks on m processors. A task is given a unique index within the range $1..n$ and a processor is given a unique index within the range $1..m$. The speed of processor p is denoted by S_p with the interpretation that if a task executes L time units on processor p, it performs $L \cdot S_p$ units of execution.

A task τ_i generates a (potentially infinite) sequence of jobs. The time when these jobs arrive cannot be controlled by the scheduling algorithm and the time of a job arrival is unknown to the scheduling algorithm before the job arrives. It is assumed that the time between two consecutive arrivals of jobs from the same task τ_i is at least T_i. We say that a job generated by τ_i finishes execution at the time when it has performed C_i units of execution. If a job finishes execution at most T_i time units after its arrival then we say that the job meets its deadline; otherwise it misses its deadline. It is assumed that $0 < C_i$ and $0 < T_i$, and that T_i and C_i are real numbers. Note that we permit $T_i < C_i$.

The scheduling algorithm is allowed to preempt the execution of a job and there is no cost associated with preemption. Migration is not permitted; when a job resumes execution after being preempted, the job must execute on the same processor as it executed on before it was preempted. Also, if any two jobs are generated by the same task then these two jobs must execute on the same processor. It is assumed that a processor can execute at most one job at a time, and a job cannot execute on two or more processors simultaneously. It is also assumed that T_i and C_i of all tasks are known to the scheduling algorithm.

Our goal is to design an algorithm that schedules tasks to meet the deadlines of all jobs. Unfortunately, the problem of deciding if a set of tasks can be partitioned such that all tasks on each processor meet deadlines is NP-complete [2]. Consequently, the problem of assigning tasks to processors is intractable. For this reason, we will allow an algorithm to fail to assign tasks to processors even when it would be possible to assign tasks to processors such that deadlines would be met. For such scheduling algorithms, it is common to characterize the performance with the notion of a *utilization bound* [12]. This notion has the additional advantage of allowing designers to find out if a specific task set will meet deadlines before run-time; this is often called *schedulability analysis*. Unfortunately, the standard definition of a utilization bound used in uniprocessor scheduling [12] and on multiprocessors with identical processors [1] cannot be applied on uniform processors. For this reason, we will instead use another performance metric: the *speed competitive ratio*.

The speed competitive ratio of an algorithm A is denoted CPT_A. It is a number such that for every task set τ and for every uniform multiprocessor system Π', characterized by its speeds $S'_1, S'_2, \ldots, S'_m$ it holds that if it is possible (using migration if necessary) to meets all deadlines of τ on Π' then algorithm A meets all deadlines of τ on Π, where Π is a uniform multiprocessor system where each processor has a speed CPT_A greater than the corresponding processor in Π'.

A low speed competitive ratio indicates high performance. A speed competitive ratio of 1 is the best achievable. And a speed competitive ratio of 2 is (as we will see) the best achievable for scheduling algorithms that do not al-

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low migration. If a scheduling algorithm has a finite speed competitive ratio then one can solve every problem instance using processors that are sufficiently fast. If no finite speed competitive ratio has been proven for a scheduling problem then one cannot know if faster processors will ever help. Unfortunately, with the current state of art in partitioned scheduling on uniform multiprocessor, there is no algorithm with a proven finite speed competitive ratio.

Therefore, in this paper we propose a partitioned scheduling algorithm for uniform multiprocessors; it allows preemption and it uses Earliest-Deadline-First (EDF) [12] on each processor. We prove its speed competitive ratio: it is at most three.

The remainder of this paper is organized as follows. Section 2 discusses design issues for uniform multiprocessors. Section 3 discusses the problem of deciding whether it is possible to schedule a task set on a uniform multiprocessor assuming that the scheduling algorithm is permitted to migrate tasks. Section 4 presents our new algorithm which does not migrate tasks. We also prove its speed competitive ratio. This proof uses results in Section 3 on scheduling where migration is permitted. Section 5 discusses the ability of previous work to solve the addressed problem. Section 6 gives conclusions.

2. Design issues

Recall that task migration is not permitted. We assume that when a job resumes execution after being preempted, the job must execute on the same processor as it executed on before it was preempted. We also assume that if any two jobs are generated by the same task, then these two jobs must execute on the same processor. This type of scheduling is called *partitioned multiprocessor scheduling* because it is equivalent to partitioning the set of tasks such that all tasks in a partition are assigned to its dedicated processor and then a uniprocessor scheduling algorithm is used at run-time. The run-time scheduling is trivial. It is well-known that preemptive Earliest-Deadline-First (EDF) is optimal on a uniprocessor with our task model; that is, it meets deadlines if there is any uniprocessor scheduling algorithm that meets deadlines. For this reason, we will, in the remainder of the paper, assume that preemptive EDF is used on each processor. For convenience we will refer to EDF with the meaning of preemptive EDF.

The problem of partitioning the task set is however non-trivial. It is important that the task assignment algorithm is aware of the scheduling algorithm used on a uniprocessor and it must use a uniprocessor schedulability test to know this. For EDF it is known [12] that:

Theorem 1. *Let p be a processor of speed S_p = 1. If $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq$ 1 and tasks are scheduled with EDF on p then all deadlines are met.*

We can easily remove the restriction S_p = 1 from Theorem 1.

Theorem 2. *Let p be a processor of speed S_p. If $\sum_{i=1}^{n} \frac{C_i}{T_i}$ $\leq S_p$ and tasks are scheduled with EDF on p then all deadlines are met.*

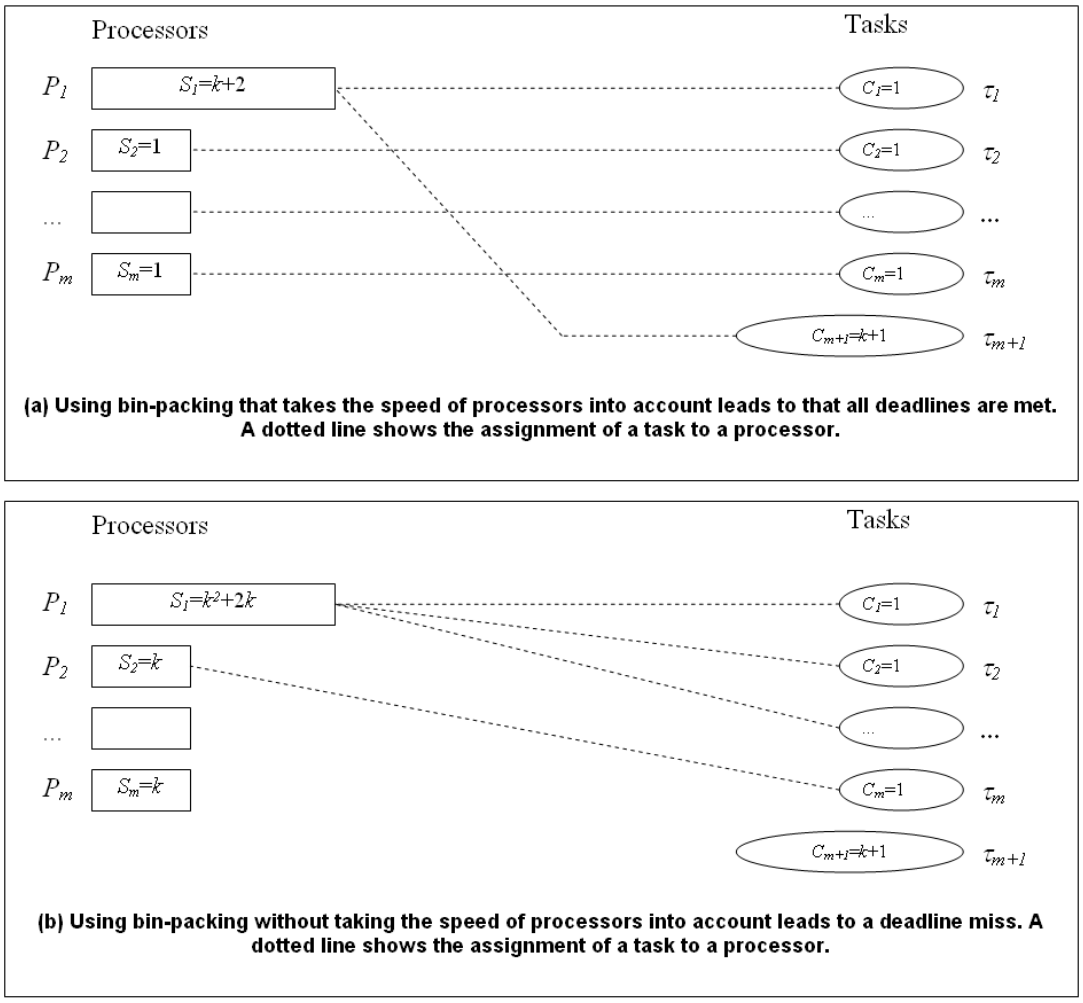
When assigning tasks to processors, the speed of a processor clearly is used in the schedulability test, for example the one in Theorem 2. But it is also important that processors are considered in the right order, in order to achieve a finite speed competitive ratio. Example 1 illustrates this.

Example 1. *Let k be an arbitrary integer such that $k \geq 2$. Consider n=k^3+1 tasks to be scheduled on m=k^3 processors. All tasks have T_i = 1. Tasks with $i \in$ 1..m, have C_i = 1 and the task m+1 has C_{m+1} = k+1. Processor 1 has the speed S_1 = k+2 and the processors with index 2..m have the speed S_p = 1.*

Observe Figure 1. It can be seen (from Figure 1a) that this task set can be scheduled by assigning τ_{m+1} to processor 1 and one of the other tasks to processor 1, and the other tasks given one dedicated processor each. However, consider Figure 1b. If the task assignment scheme considers tasks and processors in order of their index and uses a normal bin-packing algorithm, then a deadline is missed. A deadline is still missed even if processors are k times faster. We can see this as follows. Processor 1 will have the speed S_1 = k^2+2k and processors 2,3,4,. . .,m will have speed S_p = k. The speed of processor 1 is not enough to host all the tasks 1, 2, 3,. . ., m because their cumulative utilization is k^3 and this exceeds the speed of processor 1, which is S_1 = k^2+2k (it is true that $k^3 \geq k^2$+2k since $k \geq 2$). Consequently, task τ_{m+1} will not be assigned to processor 1 and hence τ_{m+1} must be assigned to one of the processors with index 2,3,. . .,m. But τ_{m+1} cannot be assigned to a processor with index 2,3,. . .,m because the utilization of τ_{m+1} is k+1 and the speed of each of the processors is k.

We have seen that algorithms using bin-packing can fail if the speed of the processors is not considered in the assignment algorithm. This can happen although these algorithms are given processors that are k times faster. We can do this reasoning for any $k \geq 2$. By letting $k \to \infty$ we obtain that the speed competitive ratio is infinite for these bin-packing schemes that do not take the speed of each processor into consideration. This stresses the importance of taking the speed of processors into account when the task assignment algorithm makes decisions.

We saw in Example 1 that it is important to take the speed of processors into account when assigning tasks to processors. In particular, if a task can be assigned to a processor such that this task occupies a large fraction of the processing capacity of that processor, then it is beneficial to assign



(a) Using bin-packing that takes the speed of processors into account leads to that all deadlines are met. A dotted line shows the assignment of a task to a processor.

(b) Using bin-packing without taking the speed of processors into account leads to a deadline miss. A dotted line shows the assignment of a task to a processor.

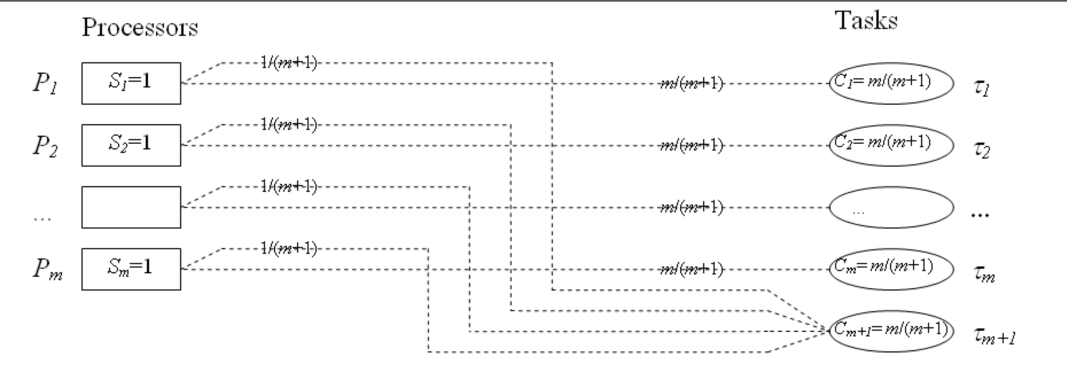
Figure 1. It is important to exploit knowledge of the speed of the processors when assigning tasks to processors. Otherwise, the speed competitive ratio can approach infinity.

the task to that processor. Considering that we will use the speed competitive ratio as a performance metric, it is interesting to find out how good performance can be achieved. Clearly, we want as low speed competitive ratio as possible, and clearly a speed competitive ratio less than 1 is impossible. But since we study scheduling with no migration, a speed competitive ratio of two is the best achievable, as it will be shown in Example 2.

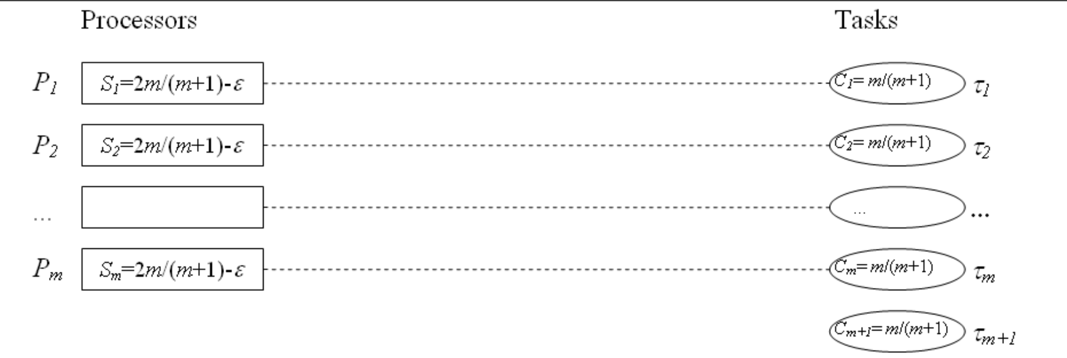
Example 2. *Observe Figure 2. Consider m+1 tasks to be scheduled on m processors. All tasks have T_i = 1, C_i = m/(m+1). All processors have speed S_p = 1. It can be seen that these tasks can be scheduled to meet deadlines with an algorithm that allows task migration because $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq m$ and all processors are identical. Figure 2a shows this. Let us now try to schedule these tasks without migration on processors of speed S_p = 2m/(m+1)-ϵ, where $\epsilon > 0$. It is necessary that two or more tasks are assigned to the same processor. On that processor, the utilization exceeds the speed of the processor and hence a deadline is missed. We can do this reasoning for any $m \geq 1$ and for any $\epsilon > 0$. Letting $m \rightarrow \infty$ and $\epsilon \rightarrow 0$ yields that a deadline is missed although the speed is arbitrarily close to two. Hence, it is impossible to achieve a speed competitive ratio less than 2 for partitioned scheduling.*

3. Optimal Scheduling With Migration

We will now discuss feasibility testing of scheduling with migration; that is, we will state conditions such that if and only if these conditions are true for a task set then it is possible to schedule the task set. We will state those conditions for a heterogeneous multiprocessor platform (in



(a) Scheduling with migration allowed. A dotted line shows the assignment of a task to a processor. The number attached to the line indicates how much computing capacity that this assignment requests from the processor.



(b) Scheduling where migration is not allowed but processors are faster. A dotted line shows the assignment of a task to a processor. There is no processor where τ_{m+1} can be assigned.

Figure 2. The speed competitive ratio of every partitioned scheduling algorithm is at least 2.

Section 3.1) and then we will state them (in Section 3.2) for uniform platforms. The latter is useful for proving the speed competitive ratio of the new algorithm in Section 4.

3.1 Heterogeneous Multiprocessor Platforms

The problem of feasibility testing on a heterogeneous multiprocessor platform has been studied previously [3]. We define $r_{i,p}$ as follows: on a heterogeneous multiprocessor platform, a task τ_i executing on processor p for L time units, performs $r_{i,p} \cdot L$ units of work. Let $x_{i,p}$ denote the fraction of time that task τ_i spends on processor p. It holds that a task set is feasible on a heterogeneous multiprocessor platform if and only if $l \leq 1$ for the following optimization problem.

minimize l

subject to:

$$\forall i \in \{1,2,\ldots,n\} : \sum_{p=1}^{m} x_{i,p} \cdot r_{i,p} = \frac{C_i}{T_i}$$

$$\forall i \in \{1,2,\ldots,n\} : \sum_{p=1}^{m} x_{i,p} \leq l$$

$$\forall p \in \{1,2,\ldots,m\} : \sum_{i=1}^{n} x_{i,p} \leq l$$

3.2 Uniform Multiprocessor Platforms

We can specialize the feasibility analysis in Section 3.1 to uniform multiprocessors. We have $\forall p$: $r_{1,p}$ = $r_{2,p}$ = $r_{3,p}$ = . . . = $r_{n,p}$ = S_p, where S_p is the speed of processor p and $r_{i,p}$ is the parameter from Section 3.1.

Let us substitute $x_{i,p} \cdot S_p$ with $u_{i,p}$. Then, the feasibility test can then be reformulated as follows: A task set is feasible on a uniform multiprocessor platform if and only if $l \leq 1$ for the following optimization problem.

minimize l
subject to:

$$\forall i \in \{1,2,\ldots,n\} : \sum_{p=1}^{m} u_{i,p} = \frac{C_i}{T_i} \tag{1}$$

$$\forall i \in \{1,2,\ldots,n\} : \sum_{p=1}^{m} \frac{u_{i,p}}{S_p} \leq l \tag{2}$$

$$\forall p \in \{1,2,\ldots,m\} : \sum_{i=1}^{n} \frac{u_{i,p}}{S_p} \leq l \tag{3}$$

From (1), (2) and (3) we obtain Lemma 1.

Lemma 1. *If it holds that:*

$$\sum_{p=1}^{m} S_p < \sum_{i=1}^{n} \frac{C_i}{T_i}$$

then no scheduling algorithm can meet all deadlines.

Proof. We know from the assumption of the lemma that there is a task set τ and a uniform multiprocessor Π such that:

$$\sum_{p=1}^{m} S_p < \sum_{i=1}^{n} \frac{C_i}{T_i}$$

Applying (1) yields:

$$\sum_{p=1}^{m} S_p < \sum_{i=1}^{n} \sum_{p=1}^{m} u_{i,p}$$

and swapping the summation order yields:

$$\sum_{p=1}^{m} S_p < \sum_{p=1}^{m} \sum_{i=1}^{n} u_{i,p}$$

This requires that there is a p such that:

$$S_p < \sum_{i=1}^{n} u_{i,p}$$

Dividing by S_p yields:

$$1 < \sum_{i=1}^{n} \frac{u_{i,p}}{S_p}$$

Hence it is impossible to satisfy (3) and $l \leq 1$.

Consequently, a deadline will be missed. This proves the lemma. □

Algorithm 1 EDF-DU-IS-FF, a task assignment algorithm for a uniform multiprocessor.

```
1: sort processors such that $S_1 \leq S_2 \leq \ldots \leq S_m$
2: sort tasks such that $C_1/T_1 \geq C_2/T_2 \geq \ldots \geq C_n/T_n$
3: for all p in 1..$m$ do
4:    U[p] := 0
5: end for
6: i := 1
7: while (i<=n) do
8:    p := 1
9:    allocated := FALSE
10:   while (p<=m) and (allocated=FALSE) do
11:      if U[p]+ $C_i/T_i$ <=$S_p$ then
12:         assign task i to processor p
13:         U[p] := U[p]+ $C_i/T_i$
14:         allocated := TRUE
15:         i := i + 1
16:      else
17:         p := p + 1
18:      end if
19:   end while
20:   if (allocated=FALSE) then
21:      declare FAILURE
22:   end if
23: end while
24: declare SUCCESS
```

4. The new algorithm

The new algorithm is described in Algorithm 1. It is called EDF-DU-IS-FF because it uses EDF on each processor, it sorts tasks in order of Decreasing-Utilization, it sorts processors in order of Increasing-Speed and it uses First-Fit bin-packing.

Line 11 is the schedulability test from Theorem 2. It is straightforward to see that the algorithm has the time complexity $O(n \cdot m + n \log n + m \log m)$. The performance of EDF-DU-IS-FF is given by Theorem 3.

Theorem 3. $CPT_{EDF-DU-IS-FF} \leq 3$

Proof. We can prove it using contradiction. We will do so and show that a failed task set must request more than 50% of the processing capacity of a subset of processors. We will then consider this task set to be scheduled using a scheduling algorithm where migration is allowed and a computing platform with lower speed is used. It will turn out that every such migrative algorithm must utilize more than the sum of the computing capacity of the subset of processors. This will contradict Lemma 1 and it proves the theorem. Let us elaborate this reasoning.

If the theorem was false then there exists a task set TF such that EDF-DU-IS-FF declares FAILURE on multiprocessor platform Π. But if TF is to be scheduled on Π' then it is possible to meet all deadlines. It must be that on Π' a processor has a speed which is $1/x$ of the speed of its corresponding processor in Π and x >3.

Consider the situation when EDF-DU-IS-FF was given TF as input and EDF-DU-IS-FF declared FAILURE. There must have been a task $\tau_{failure}$ that was considered when EDF-DU-IS-FF declared FAILURE. We can delete all tasks with index greater than $\tau_{failure}$ and we still would have a task set such that the theorem was false. We let τ denote this task set. Clearly we have:

$$\begin{aligned} &Applying\ \tau\, on\ \Pi\ using\ EDF-DU-IS-FF \\ &\qquad declares\ FAILURE \end{aligned} \tag{4}$$

and

$$It\ is\ possible\ to\ schedule\ \tau\, on\ \Pi'\ to\ meet\ deadlines \tag{5}$$

It was task τ_n that declared failure in (4). Let k denote the number of processors such that $S_p < C_n/T_n$. Due to the sorting performed on line 1 and line 2 we obtain that:

$$\begin{aligned} &For\ every\ (p,i)\ such\ that\ p \in 1,2,\ldots,k\ and\ for \\ &\qquad every\ i \in 1,2,\ldots,n\ it\ holds\ that: S_p < C_i/T_i. \end{aligned} \tag{6}$$

From (6) it follows that:

$$\begin{aligned} &When\ EDF-DU-IS-FF\ is\ run,\ no\ tasks\ are \\ &\qquad assigned\ to\ processor\ p\ with\ p \in 1,2,..,k. \end{aligned} \tag{7}$$

Let us now consider τ_n, the task that caused failure for EDF-DU-IS-FF. We know that:

$$\begin{aligned} &For\ p \in k+1,2,\ldots,m, it\ holds\ that \\ &\qquad U[p] + C_n/T_n > S_p \end{aligned} \tag{8}$$

Observe from (8) that τ_n could not be assigned to any of the processors k+1,k+2,…,m, despite the fact that $C_n/T_n \le S_p$ for those processors. Hence we have that:

$$\begin{aligned} &When\ EDF-DU-IS-FF\ declares\ FAILURE, \\ &\quad for\ each\ processor\ p \in k+1,k+2,..,m \\ &\quad it\ holds\ that:\ there\ is\ at\ least\ one\ task \\ &\qquad assigned\ to\ processor\ p. \end{aligned} \tag{9}$$

We have that Fact 1 is true.

Fact 1. *When EDF-DU-IS-FF declares failure, it holds for $p \in k$+1,k+2,…,m: U[p] > 0.50 · S_p.*

Proof. If Fact 1 was false then there must exist a processor p with U[p] $\le$ 0.50 · S_p. We know from (9) that there is at least one task assigned to processor p. Hence there is a task with $C_i/T_i \le 0.50 \cdot S_p$ assigned to processor p. Due to the sorting of tasks we have that $C_n/T_n \le C_i/T_i$ and it leads to $C_n/T_n \le 0.50 \cdot S_p$. But then it would be possible for τ_n to be assigned to processor p and we know that it cannot happen since EDF-DU-IS-FF declared failure. This is a contradiction and it proves the fact. □

From Fact 1 we obtain that when EDF-DU-IS-FF declares failure it holds that:

$$\sum_{p=k+1}^{m} 0.5 \cdot S_p < \sum_{p=k+1}^{m} U[p] \tag{10}$$

Since τ_1, τ_2, …, τ_{n-1} were assigned, we obtain from (10) that:

$$\sum_{p=k+1}^{m} 0.5 \cdot S_p < \sum_{i=1}^{n-1} \frac{C_i}{T_i} \tag{11}$$

Let us consider two cases.

Case 1. k = 0.

We have $S'_p \le S_p/x$, where S'_p is the speed of processor p in Π'. We also have $x > 3$. Combining this with (11) yields:

$$\sum_{p=1}^{m} 0.5 \cdot 3 \cdot S'_p < \sum_{i=1}^{n-1} \frac{C_i}{T_i}$$

Simplifying the left-hand side, relaxing it and adding the utilization of τ_n to the right-hand side yields:

$$\sum_{p=1}^{m} S'_p < \sum_{i=1}^{n} \frac{C_i}{T_i} \tag{12}$$

From (12) and Lemma 1, it follows that no algorithm can schedule the task set on Π' even if migration is permitted. This contradicts (5). (End of Case 1)

Case 2. $k \ge 1$.

Let us study a migrative scheduling algorithm that meets all deadlines of τ on Π'. Hence the optimization (1)-(3) has a solution with $l \le 1$. Fact 2 and Fact 3 reason about this solution.

Fact 2. *For any i, it holds that*

$$\sum_{p=1}^{k} u_{i,p} \le S'_k$$

Proof. From (2) we obtain that in a migrative schedule where deadlines are met, it holds that:

$$\sum_{p=1}^{m} \frac{u_{i,p}}{S'_p} \leq 1$$

Taking the sum over only a subset yields:

$$\sum_{p=1}^{k} \frac{u_{i,p}}{S'_p} \leq 1$$

Using the fact that the speeds of processors are sorted in ascending order yields:

$$\sum_{p=1}^{k} \frac{u_{i,p}}{S'_k} \leq 1$$

By a simple rewriting this gives us Fact 2. □

Fact 3. *For any i, it holds that*

$$\frac{C_i}{T_i} \leq \frac{x}{x-1} \cdot \sum_{p=k+1}^{m} u_{i,p}$$

Proof. From (6) we obtain:

$$S_p < \frac{C_i}{T_i} \tag{13}$$

Based on (13) and (1) we have:

$$S_k < \sum_{p=1}^{m} u_{i,p} \tag{14}$$

From the assumption on Π and Π' we obtain:

$$S'_k < \frac{S_k}{x} \tag{15}$$

Combining Fact 2 and (15) yields:

$$\sum_{p=1}^{k} u_{i,p} \leq \frac{S_k}{x} \tag{16}$$

From (16) we obtain:

$$\sum_{p=1}^{m} u_{i,p} \leq \frac{S_k}{x} + \sum_{p=k+1}^{m} u_{i,p} \tag{17}$$

Combining (14) and (17) yields:

$$\sum_{p=1}^{m} u_{i,p} \leq \frac{\sum_{p=1}^{m} u_{i,p}}{x} + \sum_{p=k+1}^{m} u_{i,p} \tag{18}$$

Rewriting (18) and using (1) yields:

$$\frac{C_i}{T_i} \leq \frac{x}{x-1} \cdot \sum_{p=k+1}^{m} u_{i,p}$$

□

Recall from (11) that when we used partitioning we had:

$$\sum_{p=k+1}^{m} 0.5 \cdot S_p < \sum_{i=1}^{n-1} \frac{C_i}{T_i}$$

Applying Fact 3 yields:

$$\sum_{p=k+1}^{m} 0.5 \cdot S_p < \frac{x}{x-1} \cdot \sum_{i=1}^{n-1} \sum_{p=k+1}^{m} u_{i,p}$$

We have $S'_p \leq S_p/x$, where S'_p is the speed of processor p in Π'. Applying this yields:

$$\sum_{p=k+1}^{m} 0.5 \cdot x \cdot S'_p < \frac{x}{x-1} \cdot \sum_{i=1}^{n-1} \sum_{p=k+1}^{m} u_{i,p}$$

Rewriting yields (and using the knowledge that x is positive) yields:

$$\sum_{p=k+1}^{m} S'_p < \frac{2}{x-1} \cdot \sum_{i=1}^{n-1} \sum_{p=k+1}^{m} u_{i,p}$$

Since $x > 3$ we obtain that 2/(x-1) $<$ 1. Using it yields:

$$\sum_{p=k+1}^{m} S'_p < \sum_{i=1}^{n-1} \sum_{p=k+1}^{m} u_{i,p}$$

Swapping the order of the indices of the summation on the right-hand side yields:

$$\sum_{p=k+1}^{m} S'_p < \sum_{p=k+1}^{m} \sum_{i=1}^{n-1} u_{i,p}$$

This requires that there is a p such that:

$$S'_p < \sum_{i=1}^{n-1} u_{i,p}$$

Dividing by S'_p yields:

$$1 < \sum_{i=1}^{n-1} \frac{u_{i,p}}{S'_p}$$

And hence it is impossible to satisfy (3) and $l \leq 1$. Consequently, a deadline will be missed on Π'. But this contradicts (5). (End of Case 2)

We can see that regardless of the case, we obtain a contradiction and hence Theorem 3 is true. □

5. Previous work

Algorithms in operations research have been proposed for scheduling jobs with no real-time requirements assuming that all jobs arrive at the same time and the goal is to minimize the time when all jobs have been finished. (See for example [10].) A solution to this problem can be used for scheduling periodically arriving tasks with deadlines [2]. But unfortunately, that algorithm [2] allows task migration and hence it cannot solve our problem.

The problem of partitioning a task set on a uniform multiprocessor has been considered previously [8, 7]. This is the same problem as we addressed in this paper. We find two drawbacks with those algorithms and analysis though. First, the algorithms are analyzed by extending the utilization bound from identical multiprocessors. But their utilization bound is not a single number; it is a function of the maximum C_i/T_i of tasks. This causes a large amount of pessimism when (i) the difference in speeds of processors is very large and (ii) the maximum C_i/T_i is large. This pessimism is neither a consequence of the algorithm, nor the analysis techniques but it is a consequence of the definition of the utilization bound in uniform multiprocessors. The second drawback of the above mentioned previous work [8, 7] is that their speed competitive ratio is infinite. The algorithms use First-Fit or Any-Fit; this is a good design. However, the algorithms sort processors in increasing order of speed; this makes it possible for the behavior of Example 1 to occur and it causes the speed competitive ratio to be infinite.

The problem we address can be solved using task assignment algorithms for heterogeneous multiprocessors [5, 4]. The algorithm in [5] uses exhaustive enumeration of "heavy tasks" and this leads to a time complexity of O(m^m). The other algorithm [4] has polynomial time-complexity but it is high; it requires that a linear program is solved. Neither of them proves a speed competitive ratio.

A speed competitive ratio has already been proven for scheduling real-time tasks on uniform multiprocessors [9, 6]; one of the algorithms has a speed competitive ratio of two [9]. In addition it has the advantage of being proven not just for the sporadic task model but for the more generic model of aperiodic jobs where the scheduling algorithm has no knowledge of jobs arriving in the future. Unfortunately, it requires that tasks can migrate.

We have studied uniform multiprocessors and we studied how much extra processing power must be given to our algorithm to ensure that our algorithm meets deadlines for every task set which an optimal algorithm meets deadlines. This type of analysis was originally proposed in [11, 13] but for a uniprocessor [11] and a multiprocessor with where all processors have the same speed [13].

6. Conclusions

We have presented an algorithm to schedule sporadically arriving tasks on a uniform multiprocessor and we have proven its speed competitive ratio. It is at most three. This is a significant result because it is the first proven speed competitive ratio in real-time scheduling on uniform multiprocessors where migration is not allowed.

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